

An Earlier Date for “Cramer’s Rule”

Bruce A. Hedman

The University of Connecticut at Hartford, 85 Lawler Road, West Hartford, Connecticut 06117-2697

Carl B. Boyer has shown that Colin Maclaurin published “Cramer’s Rule” two years before Gabriel Cramer and conjectured that Maclaurin knew the technique as early as 1729. This note establishes Boyer’s conjecture through a heretofore unpublished manuscript in the Edinburgh University Library. © 1999 Academic Press

Carl B. Boyer démontre que Colin Maclaurin publia “la Règle de Cramer” deux ans avant Gabriel Cramer, et spécule que Maclaurin fût en possession de la technique dès 1729. Cette note-ci établit la conjecture de Boyer grâce à un manuscrit qui se trouve dans la Bibliothèque de l’Université d’Edimbourg et qui est jusqu’ici inédit. © 1999 Academic Press

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In 1750, Gabriel Cramer (1704–1752) published the technique for solving a simultaneous system of equations which would come to bear his name [2]. Carl B. Boyer [1] showed that, in fact, “Cramer’s Rule” was published two years earlier in Colin Maclaurin’s posthumous *Treatise of Algebra*, and suggested that the rule instead be called the “Maclaurin–Cramer Rule.” Boyer conjectured that Maclaurin knew this rule as early as 1729, based on a letter Maclaurin [4] contributed to the *Philosophical Transactions of the Royal Society* in which he claimed then to be writing his *Treatise of Algebra*.

His letter of 1729 neither explicitly gives “Cramer’s Rule” nor implicitly refers to it, however. An unpublished manuscript of 1729 *does* confirm Boyer’s conjecture. During the fall of 1998, I was on sabbatical leave at the University of Edinburgh to study their collection of Maclaurin’s unpublished manuscripts. This includes a handwritten copy of Maclaurin’s *Treatise of Algebra* [5] copied by John Russell in Edinburgh and dated 1729. It consists of 52 leaves bound in full calf. The introduction promises a treatise of three parts, but only Part I dealing with the general rules of algebra is copied down on folio pages numbered through 76. Then follow four blank leaves, then three pages of a series derivation of π , and then a one-page index. The earliest published reference to the Russell manuscript I have found is by Erik Sageng [6, 32]. A microfilm copy of the Russell manuscript is now included in the Homer Babbidge Library of the University of Connecticut.

Part I is divided into 14 chapters almost identical to the posthumous 1748 edition. Chapter 12, entitled “Containing General Theorems for Exterminating the Unknown Quantities in Given Equations,” contains on folio pp. 65 and 66 the same form of “Cramer’s Rule” as the 1748 edition (see Figs. 1 and 2). Apparently, Maclaurin circulated working copies of his manuscript among his students years before he brought it into publishable form.

65.

S. 3. *THEOREM 2.* Suppose now that 3 unknown quantities x, y, z and these:

$$ax + by + cz = m$$

$$dx + ey + fz = n$$

$$gx + hy + kz = p$$

$$\text{Then shall } z = \frac{aep - ahn + dhm - dbp + gbn - gec}{ahk - ahf + dhe - dbk + gbf - gec}$$

where the numerator consist of all the products can be made of three opposite taken out of the orders in which z and the Denominator consists of all the can be made of three opposite Coefficients of the orders that involve the three unknown quantities.

For from the first it appears That

$$y = \frac{an - afz - dm + dez}{ae - db}$$

$$\text{And also That } y = \frac{ap - akz - gm - gec}{ah - gb}$$

$$\text{Therefore } \frac{an - afz - dm - dez}{ae - db} = \frac{ap - akz - gm - gec}{ah - gb}$$

$$\text{Therefore } \frac{an - afz - dm + dez}{ae - db} \times ah - gb \times \frac{an - afz + gba}{ae - db}$$

$$= \frac{ap - gm - akz + gec}{ah - gb} \times ae - db \times \frac{ap - nkz + gbdm}{ah - gb}$$

gbdm - gbd - z from both sides and the

FIG. 1. Page 65 of the 1729 manuscript of Maclaurin's *Treatise of Algebra*. Edinburgh University Library Shelfmark Dc. 3.66. I thank the Library for permission to publish this as well as Fig. 2.

Thus, the Russell manuscript provides conclusive evidence that Maclaurin was teaching his students "Cramer's Rule" over 20 years before Cramer published it.

Maclaurin's life was brought to an untimely end through his exertions to defend the city of Edinburgh against the Jacobites [3]. He thus was prevented from bringing his *Treatise of*

The Values of x and y are found after the Same
and have the Same Denominator; For Exam.

$$y = \frac{ap - akn + dhm - dep + gen - gfm}{aik - afh + dke - dbk + gbf - gec}$$

§. 4. If any term is wanting in any of the
given Equations, the Value of x and y , will be
more simple; Suppose for Example that f are
equal to nothing, Then the term fz will van-
the. Second Equation and kz in the third; and

$$= \frac{ap - ank + dhm - dbp + gbm - gem}{dke - gec}$$

and $y = \frac{gen - gep}{dke - gec}$

§. 5. If four Equations are given involv-
unknown quantities, their Values may be found
after the Same manner. By taking all the pr-
can be made of four opposite Coefficients an-
prefixing contrary Signs to those that involv-
=ducts. of two opposite Coefficients.

Chap: 13. Of Quadratick Equa-

§. 1. In this Solution of any question, the
have not an Equation that involves only one

FIG. 2. Page 66 of the 1729 manuscript of Maclaurin's *Treatise of Algebra*. Edinburgh University Library Shelfmark Dc. 3.66 (by permission).

Algebra into final form. To benefit his widow and young family, Patrick Murdoch prepared it for posthumous publication in 1748. It became one of the most popular algebra texts in Britain during the 18th century, running into a sixth edition at London in 1796. As Boyer suggests, perhaps it was Cramer's more concise notation that popularized this rule, so that

it came to bear his name, rather than that of the much-loved professor who taught it to his students a generation before.

REFERENCES

1. Carl B. Boyer, Colin Maclaurin and Cramer's Rule, *Scripta Mathematica* **27** (1966), 377–379.
2. Gabriel Cramer, *Introduction à l'analyse des lignes courbes algébriques*, Geneva, 1750, pp. 657–659.
3. Bruce Hedman, An Annotated Edition of Maclaurin's Journal Regarding the '45, submitted for publication.
4. Colin Maclaurin, A Second Letter from Mr. Colin Maclaurin Concerning the Roots of Equations, with the Demonstration of Other Rules in Algebra, *Philosophical Transactions of the Royal Society of London* **36** (1729), 59–96.
5. Colin Maclaurin, *A Treatise of Algebra by Mr. Colin Maclaurin, Professor of Mathematicks in the University of Edinburgh*, written by John Russell, Edinburgh, 1729. [Edinburgh University Library, manuscript Dc. 3.66]
6. Erik Lars Sageng, *Colin Maclaurin and the Foundations of the Method of Fluxions*, Ph.D. thesis, Princeton University, 1989.