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Reviewed work(s):
Source: *The College Mathematics Journal*, Vol. 31, No. 4 (Sep., 2000), pp. 286-289
Published by: Mathematical Association of America
Stable URL: http://www.jstor.org/stable/2687418
Accessed: 27/10/2012 18:47

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Colin Maclaurin's Quaint Word Problems

Bruce Hedman

Bruce Hedman was first inspired to pursue research in mathematics by Branko Grunbaum of the University of Washington, where he did his undergraduate work. He took his Doctorate under Harold Kuhn at Princeton University in 1979, and has been on faculty in the mathematics department of the University of Connecticut since 1982. He is also an ordained Congregational minister. He spent a sabbatical leave at the University of Edinburgh, and he and his wife Sandy perform semi-professionally singing traditional Scottish and Irish folk music.

Those of us who earn our daily bread teaching algebra keep an eye out for unusual problems to illustrate routine methods. In 1998, I was on sabbatical leave at the University of Edinburgh researching some unpublished manuscripts of Colin Maclaurin (1698–1746), after whom the Maclaurin series is named. The Special Collection of the Edinburgh University Library includes "Ane Introduction to the Mathematicks" **[1]**, in Maclaurin's own handwriting. This work poses some quaint elementary problems that should intrigue today's students.

Maclaurin was hired by the University of Edinburgh in 1725 to take over the aging James Gregory's teaching duties. From a letter to Martin Folkes **[2]** we know that Maclaurin had begun drafting his *Treatise of Algebra* as early as 1726. This would become Great Britain's most popular textbook on algebra until well into the nineteenth century. Like modern authors, Maclaurin developed his book by using a manuscript form in his classes. A copy of Part I, in a student's handwriting, survives (dated 1729) **[3]**. A later manuscript, again in a student's handwriting, includes all three parts of the final version, and dates to around 1740. Maclaurin died before his *Treatise of Algebra* could be published in its final form. His widow and five young children were left in financial difficulties. A friend, Patrick Murdoch, was given the manuscripts to edit for publication, and it was finally printed in London in 1748.

When Maclaurin came to Edinburgh, he discovered that not all of his students were prepared for the *Treatise*. Then as now, some students needed a review of what we call high school algebra. To meet this need Maclaurin wrote "Ane Introduction to Mathematicks," which, like our modern Schaum's Outlines, was full of word problems, worked out in complete detail.

Erik Sageng **[4]** regards the text as merely a student's notes on a lecture course, but I disagree. Maclaurin's handwriting is distinctively round. The details of each problem are carefully worked out at length without mistakes. It begins with a well-honed preface about the nature of mathematics. "Ane Introduction" is written on 43 pages of 153 leaves bound in full calf; the other side of the notebook contains a philosophical treatise dated 1725, also in Maclaurin's handwriting. I presume Maclaurin cannibalized an old notebook when he came to Edinburgh, so as to circulate a primer among his students.

At first, Maclaurin's word problems struck me as very familiar: two pipes filling a vessel at different rates, two ships approaching at different speeds, two painters working together. I mused on how little has changed in algebra over two and a half

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centuries. Then, as Maclaurin's problems grow more difficult, the thought dawned on me that in fact these exercises would challenge today's students considerably. In what follows I have modernized the spelling and vocabulary, as indicated by brackets.

Maclaurin, also a classical scholar, posed this ancient Greek problem, which he headed "The Ptolemaic Riddle":

> I am a bronze lion. [Out of] my mouth, the sole of my right foot, and my two eyes [come four] pipes that fill a system in differenttimes. The right eye fills it in two days, the left in three, the sole of my foot in four, but my mouth takes six days to fill it. Find in how many days all these will fill it together. [**1**, p. 21]

To the best of my knowledge selections from "Ane Introduction" have never appeared in print before now.

Today we do not wish to encourage alcoholic consumption among our students. However, in Maclaurin's day the following was an ordinary rate problem taken from daily experience:

> A man and his wife usually [drink a keg] of beer in 12 days. They found by often experience that [when his] wife [was] absent the man drank it in 20 days. In how many days will the wife alone drink it? [**1**, p. 32]

From the eighteenth century through the Napoleonic and Crimean wars British infantry were deployed in square formation to repel cavalry attack. The next problem is set in that historical context:

> A general [disposes] his army into square battle [formation]. [He] finds he has 284 soldiers [left] over and above. But increasing each side [of the square] by one soldier he [lacks] 25 to fill up the [new] square. How many soldiers [does the general have]? [**1**, p. 34]

Maclaurin defended mathematical education at universities because of its practical applications, not the least of which was in economics. Maclaurin was admired by Adam Smith (1727–1790), who wrote *The Wealth of Nations* during this period of the Scottish Enlightenment. Maclaurin supplied the initial acturial calculations which launched the Scottish Widows Fund and Assurance Society, a company still in business. In his primer he posed this basic economic problem:

> A gentleman hired a workman [on these conditions]: For every day he [worked] he [earned] 12 shillings. For every day he sat idle he forfeited 8 shillings. At the end of 390 days there was nothing [left to be paid the workman]. How many days [did he work] and how many days [did] he sit idle? [**1**, p. 35]

Did the moralistic tone of this problem reflect the prevailing work-ethic of the Scottish Enlightenment?

Naturally Maclaurin included those questions about the relative ages of parents and children that have tormented students since Ahmes the Scribe:

> A son asked his father how old he was. His father answered him thus. If you take away 5 from my years, and divide the remainder by 8, the quotient will be 1/3 of your age. But if you add 2 to your age and multiply the whole by 3, and then subtract it from the product you will have the same number [as the] years of my age. [**1**, p. 37]

Are those question eternal because in every age students can't get a straight answer out of dad?

Perhaps,a fourth of the problems Maclaurin posed in his primer were in Latin, then just declining as the lingua franca of universities. Perhaps some modern students today who enjoyed high school Latin might like this problem:

> Maritus, uxor, et filius habent annos 96, ita ut anni Mariti et filii, simul faciant annos uxoris + 15. Sed uxoris cum filii faciant Mariti + 2. [**1**, p.39]

That is:

> Maritus, his wife, and son have [combined ages of] 96 years. But Maritus and his son together make 15 years more than his wife's age. But his wife's age with their son's make 2 years more than Maritus's age. [Author's translation]

Notice that this problem involves the simultaneous solution of three equations in three unknowns. In "Ane Introduction," Maclaurin solves it by simple substitution. However, in the earliest extant manuscript of the *Treatise* only four years later, Mclaurin presents the earliest surviving version of Cramer's rule [**3**, pp. 65, 66]. In fact, this rule properly should be named for Maclaurin, who taught it to students some twenty years before Cramer's publication [**5**, **6**].

To learn more about Colin Maclaurin, who too often is given short shrift by modern historians, see Judith Grabiner's splendid paper [**7**].

References

1. Edinburgh University Library, manuscript 2651.
2. Stella Mills (ed.), *The Collected Letters of Colin Maclaurin*, Nantwich, 1982, letter 129, p. 190.
3. Edinburgh University Library, document 3.66.
4. Erik Sageng, *Colin Maclaurin and the Foundations of the Method of Fluxions*, Ph.D. thesis, Princeton, 1989.
5. C. B. Boyer, Colin Maclaurin and Cramer's Rule, *Mathematica* 27 (1966) 377–379.
6. Bruce Hedman, An Earlier Date for Cramer's Rule, *Historia Mathematica*, 26 (1999) 365–368.
7. Judith Grabiner, Was Newton's calculus a dead end? *American Mathematical Monthly*, 104:5 (1997) 393–410.

Appendix: Solutions to Maclaurin's Problems

The Ptolemaic riddle:

$$1/2 + 1/3 + 1/4 + 1/6 = 1/x, \ x = 4/5 \text{ days working together.}$$

Drinking the Keg:

$$1/20 + 1/x = 1/12, \ x = 30 \text{ days for the wife alone.}$$

Infantry Square:

$$x^2 + 284 = (x + 1)^2 - 25,$$

$x = 154$ soldiers on a side; total number of soldiers $= 154 \times 154 = 23{,}716$.

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Workman's Wages:

$$12x - 8(390 - x) = 0,\ x = 156 \text{ working days.}$$

Father's and Son's Ages:

$$(x-5)/8 = y/3,\ 3(y+2) - y = x,\ x = 9, \text{ father's age } y = 3/2, \text{ son's age [sic].}$$

Roman Family's Ages:

$$x + y + z = 96$$
$$x + z = 15 + y$$
$$y + z = x + z;$$
$$x = 47, \text{ Maritus' age, } y = 40\tfrac{1}{2}, \text{ wife's age, } z = 8\tfrac{1}{2}, \text{ son's age.}$$

Mathematics in the Media

An odd advertisement for Office.com that appeared, among other places, in *PC Magazine* (19 (2000) #11 (June 6, 2000), 209) has a picture of a sloppily-dressed youth, apparently with two-toned hair, writing mathematics on the back of a truck. Here, line for line, is what he had written:

For all $\chi(\eta)$, $\chi(\eta) \le \overline{\gamma^*\chi(\eta)}$
such that $p^*\chi(\eta+1) \le \gamma^*p^*\chi$
$p^*y(\eta) \le (\gamma) - \delta)p^*\chi(\eta)$ for
$d[\chi(\eta), \chi^*] \ge \varepsilon$ and p^*
$\chi(\eta+1) < \gamma^*p$ [symbols obscured by writer's head] (η) &
$p^*y(\bar{N}) <$

Isn't that good? It could be better, of course—unbalanced parenthesis are not desirable—but it is far superior to the jumble of meaningless symbols, usually prominently featuring square-root signs, that copy writers produce when called on to imitate mathematics.

It was not clear to me exactly what it was that Office.com was selling, but if I am ever in the market for whatever it is, I will be favorably disposed to the firm.